SET - 1

SAHODAYA PRE-BOARD EXAMINATION – 2024-25

CLASS – X

SUB: MATHEMATICS BASIC(241)

MARKING SCHEME

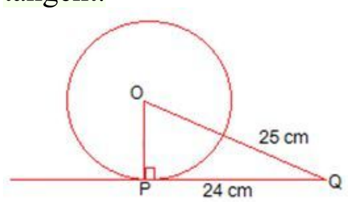
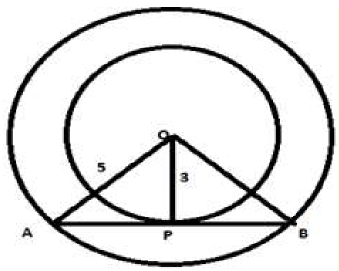
Maximum Marks : 80

NOTE:

The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. Any other alternative method is acceptable. Proportional marks are to be awarded.

	SECTION A **Q. No. 1 to 20 are Multiple Choice Questions of 1 mark each.**	
1	(A) 1	**1**
2	(A) Consistent with unique solution	**1**
3	(A) 2	**1**
4	(D) −9	**1**
5	(C) 5 cm	**1**
6	(D) 5.6cm	**1**
7	(C) 30°	**1**
8	(D) 35 cm	**1**
9	(C) $a^3 b^2$	**1**
10	(B) 137	**1**
11	(C) $\frac{55}{3}$ cm	**1**
12	(B) 60^0	**1**
13	(D) $\frac{1}{2}$	**1**
14	(A) 6 : π	**1**
15	(B) 1	**1**
16	(C) 16	**1**
17	(D) 13	**1**
18	(D) intersecting at (a, b)	**1**
19	(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	**1**
20	(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)	**1**

	SECTION B **Q. No. 21 to 25 are Very Short Answer Questions of 2 marks each.**	
21	(A) Points are collinear if sum of any two of distances is equal to the distance of the third. Let A(-1, -1), B (2, 3), C (8, 11) A, B and C are collinear if AB + BC = AC $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{3^2 + 4^2}$ $= \sqrt{9+16} = \sqrt{25} = 5$ units $BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{6^2 + 8^2}$ $= \sqrt{36+64} = \sqrt{100} = 10$ units $CA = \sqrt{(8+1)^2 + (11+1)^2} = \sqrt{9^2 + 12^2}$ $= \sqrt{81+144} = \sqrt{225} = 15$ units From above, we can see that AB + BC = 5 +10 = 15 = AC Therefore, A, B and C are collinear. **OR** (B) Let the points be A(-3, 4), B(2, 5) and C(*x,* 0) $AC = \sqrt{(-3-x)^2 + (4-y)^2}$ $= \sqrt{(-3-x)^2 + (4-0)^2}$ $AC = \sqrt{(-3-x)^2 + (4)^2}$ $BC = \sqrt{(2-x)^2 + (5-y)^2}$ $= \sqrt{(2-x)^2 + (5-0)^2}$ $BC = \sqrt{(2-x)^2 + (5)^2}$ We know that both these distances are the same. So equating both these we get, AC=BC $\sqrt{(-3-x)^2 + (4)^2} = \sqrt{(2-x)^2 + (5)^2}$ Squaring on both sides we have, $(-3-x)^2 + (4)^2 = (2-x)^2 + (5)^2$ $9 + x^2 + 6x + 16 = 4 + x^2 - 4x + 25$ $10x = 4$ $x = \frac{2}{5}$ Hence the point on the x-axis which lies at equal distances from the mentioned points is $\boxed{\left(\frac{2}{5}, 0\right)}$.	 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

22	(A) The line drawn from the centre of the circle to the tangent is perpendicular to the tangent.  $\therefore OP \perp PQ$ Also, Δ OPQ is right angled. OQ = 25 cm and PQ = 24 cm (Given) By Pythagoras theorem in Δ OPQ, $OQ^2=OP^2+PQ^2$ $\Rightarrow (25)^2=OP^2+(24)^2$ $\Rightarrow OP^2=625-576$ $\Rightarrow OP^2=49$ $\Rightarrow OP=7$ cm The diameter = 2 × 7 = 14 cm **OR** (B)  Let the two concentric circles with centre O. AB be the chord of the larger circle which touches the smaller circle at point P. $\therefore$ AB is tangent to the smaller circle to the point P. $\Rightarrow OP \perp AB$ By Pythagoras theorem in ΔOPA, $OA^2=AP^2+OP^2$ $\Rightarrow 52=AP^2+32$ $\Rightarrow AP^2=25-9$ $\Rightarrow AP = 4$ cm In ΔOPB, Since $OP \perp AB$, AP = PB (Perpendicular from the centre of the circle bisects the chord) AB = 2AP = 2 × 4 = 8 cm $\therefore$ The length of the chord of the larger circle is 8 cm.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
23	$a= -7$ $d=5$ Then its 18th term $a_n= a+[n-1]d$ $a_{18}= -7+[18-1]5$ $a_{18}= -7+17\times5$ $a_{18}= -7+85$ $a_{18}= 78$	1/2 1/2 1

24	$\tan\theta = \frac{3}{4}$, $\cot\theta = \frac{4}{3}$ $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = (\frac{4}{3})^2 = \frac{16}{9}$	1 1
25	So, modal class = 35 – 45 Lower limit (l) of modal class = 35 Frequency (f) of modal class = 23 Class size (h) = 10 Frequency (f_1) of class preceding the modal class = 21 Frequency (f_2) of class succeeding the modal class = 14 Mode $= l + \frac{f - f_1}{2f - f_1 - f_2} \times h$ $= 35 + \frac{23-21}{2 \times 23 - 21 - 14} \times 10$ $= 35 + \frac{2}{46-35} \times 10$ $= 35 + 1.81$ $= 36.81$ Clearly, the mode is 36.81. It represents that the maximum number of patients admitted in the hospital was of age 36.81 years.	$\frac{1}{2}$ $\frac{1}{2}$ 1
	SECTION C **Q. No. 26 to 31 are Short Answer Questions of 3 marks each.**	
26	Let us assume that $3+5\sqrt{2}$ is a rational number. Thus, $3+5\sqrt{2}$ can be represented in the form of $\frac{p}{q}$, where p and q are integers, q ≠ 0, p and q are co-prime numbers. $3+5\sqrt{2} = \frac{p}{q}$ $\Rightarrow 5\sqrt{2} = \frac{p}{q} - 3$ $\Rightarrow 5\sqrt{2} = \frac{p-3q}{q}$ $\Rightarrow \sqrt{2} = \frac{p-3q}{5q}$ Since, $\frac{p-3q}{5q}$ is rational $\Rightarrow \sqrt{2}$ is rational But,it is given that $\sqrt{2}$ is an irrational number. Therefore, our assumption is wrong. Hence, $3+5\sqrt{2}$ is an irrational number.	1 $\frac{1}{2}$ 1 $\frac{1}{2}$

27	Let the point (0, y) intersects the line segment joining the point (4, -5) and (-1, 2) in the ratio m : n. Using Section formula, we have. $x = \frac{mx_2 + nx_1}{m+n}$; $y = \frac{my_2 + ny_1}{m+n}$ $x = \frac{mx_2 + nx_1}{m+n}$; $0 = \frac{m\times(-1) + n\times 4}{m+n}$ $0 = \frac{-m + 4n}{m+n}$ $-m + 4n = 0$ $-m = -4n$ $\frac{m}{n} = \frac{4}{1}$ m : n = 4 : 1 This implies, $y = \frac{my_2 + ny_1}{m+n}$ $= \frac{4\times 2 + 1\times(-5)}{4+1}$ $= \frac{8-5}{5}$ $= \frac{3}{5}$ The ratio of division is 4:1 and the Coordinates of the point of division is $(0, \frac{3}{5})$.	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
28	(A) We have, LHS $= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ $= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$ $= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$ $= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$ $= \frac{2}{\cos\theta}$ $= 2\sec\theta$ = RHS. **OR**	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	(B) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$	1
	$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$	
	$= \frac{\tan\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{2\cos^2\theta - \sin^2\theta - \cos^2\theta}$	1
	$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$	
	$=\tan\theta$	1
29	Cumulative frequency table for the given data is as follows :	

Classes	Number of students (f_i)	Cumulative frequency ($c.f.$)
0-10	2	2
10-20	12	2 + 12 = 14
20-30	22	14 + 22 = 36
30-40	8	36 + 8 = 44
40-50	6	44 + 6 = 50
Total	$\sum f_i = 50$	

1

Here, $n = 50 \Rightarrow \frac{n}{2} = \frac{50}{2} = 25$

$\frac{1}{2}$

Cumulative frequency just greater than 25 is 36 and corresponding interval is 20-30.

$\therefore$ Median class is 20-30.

So, $l = 20, f = 22, c.f. = 14, h = 10$

$\frac{1}{2}$

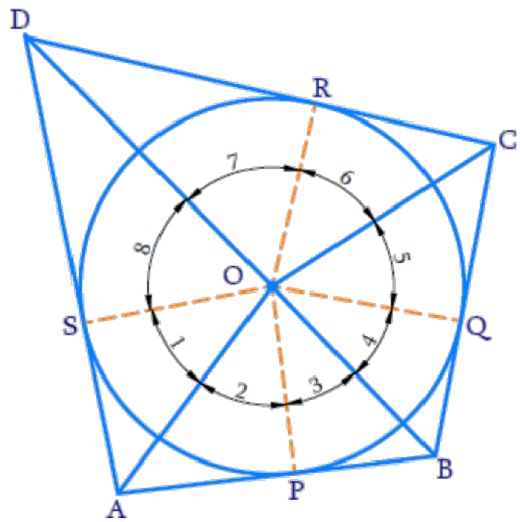
$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$

$= 20 + \left(\frac{25-14}{22}\right) \times 10 = 20 + \frac{11}{22} \times 10 = 20 + 5 = 25$

1

30

(A) We know that tangents drawn from a point outside the circle, subtend equal angles at the centre.



$\frac{1}{2}$

In the above figure, P, Q, R, S are points of contact

AS = AP (The tangents drawn from an external point to a circle are equal.)

$\angle SOA = \angle POA = \angle 1 = \angle 2$ (Tangents drawn from a point outside of the circle, subtend equal angles at the centre)

Similarly,

$\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$ $\frac{1}{2}$

Since complete angle is 360° at the centre,

We have,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$2\,(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$

(or) $2\,(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$ $\frac{1}{2}$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

(or) $\angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

From above figure,

$\angle 1 + \angle 8 = \angle AOD$, $\angle 4 + \angle 5 = \angle BOC$

and $\angle 2 + \angle 3 = \angle AOB$, $\angle 6 + \angle 7 = \angle COD$ 1

Thus we have,

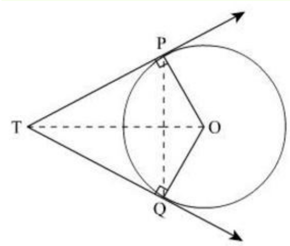
$\angle AOD + \angle BOC = 180°$

(or) $\angle AOB + \angle COD = 180°$

$\angle AOD$ and $\angle BOC$ are angles subtended by opposite sides of quadrilateral circumscribing a circle and the sum of the two is 180°. $\frac{1}{2}$

Hence proved.

OR

	(B)  We know that, the lengths of tangents drawn from an external point to a circle are equal. ∴TP=TQ In ΔTPQ TP = TQ ⇒∠TQP=∠TPQ……(1) (In a triangle, angles opposite to equal sides are equal) $\angle TQP+\angle TPQ+\angle PTQ=180^0$ (Angle sum property) ∴$2\angle TPQ+\angle PTQ=180^0$ (Using (1)) ⇒$\angle PTQ=180^0-2\angle TPQ$……(1) We know that, a tangent to a circle is perpendicular to the radius through the point of contact. OP⊥PT ∴$\angle OPT=90^0$ ⇒$\angle OPQ+\angle TPQ=90^0$ ⇒$\angle OPQ=90^0-\angle TPQ$ ⇒$2\angle OPQ=2(90^0-\angle TPQ)=180^0-2\angle TPQ$……(2) From (1) and (2), we get ∠PTQ=2∠OPQ	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
31	Let the tens digit = x Unit digit = y Original number = $10x + y$ When digits are reversed the number is $10y + x$ $x=3y$................................(i) $(10x+y)-(10y+x)=54$................................(ii) $10x+y-10y-x=54$ $9x-9y=54$ Cancelling 9 from both sides, we get, $x - y = 6$...................(iii) Now, substituting the value of x from equation (i) in equation (iii), we get, $3y - y = 6$ $2y = 6$ ∴$y = 3$ Substituting the value of y in equation (i), we get,	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

	$x = 3 \times 3$ $\therefore x = 9$ Hence, the two digit number is $10x + y = 93$.	$\frac{1}{2}$
	SECTION D **Q. No. 32 to 35 are Long Answer Questions of 5 marks each.**	
32	(A) Let the average speed of the passenger train be x km/h The average speed of the express train will be (x + 11) km /h As we know that, Distance = Speed × Time Time = Distance / speed Therefore, time taken by the passenger train to travel 132 km = 132 / x Time taken by the express train to travel 132 km = 132 / (x + 11) The difference between the time taken by the passenger and the express train is 1 hour. Therefore, we can write: $\frac{132}{x} - \frac{132}{(x+11)} = 1$ Solving $\frac{132}{x} - \frac{132}{(x+11)} = 1$ by taking the LCM on the LHS: $\frac{132(x+11) - 132x}{x(x+11)} = 1$ $\frac{132x + 1452 - 132x}{x^2 + 11x} = 1$ $1452 = x^2 + 11x$ $x^2 + 11x - 1452 = 0$ By comparing $x^2 + 11x - 1452 = 0$ with the general form of a quadratic equation $ax^2 + bx + c = 0$, we get a = 1, b = 11, c = - 1452 $b^2 - 4ac = 11^2 - 4(1)(-1452)$ $= 121 + 5808$ $= 5929 > 0$ $b^2 - 4ac > 0$ Therefore, real roots exist. $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$ $= (-11 \pm \sqrt{5929}) / 2$ $= (-11 \pm 77) / 2$ $x = (-11 + 77) / 2$ and $x = (-11 - 77) / 2$ $x = 66 / 2$ and $x = -88 / 2$ $x = 33$ and $x = -44$ x can't be a negative value as it represents the speed of the train. Thus, speed of the passenger train = 33 km/hr Speed of the express train = x + 11 = 33 + 11 = 44 km/hr	$\frac{1}{2}$ 1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

OR

(B) Let one pipe fills the cistern in x mins.

Therefore, the other pipe will fill the cistern in (x+3) mins. $\frac{1}{2}$

Time taken by both, running together, to fill the cistern = $3\frac{1}{13}$ mins = $\frac{40}{13}$ mins

Part filled by one pipe in 1 min = 1/x Part filled by the other pipe in 1 min = $\frac{1}{(x+3)}$ 1

Part filled by both pipes, running together, in 1 min = $\frac{1}{x}+\frac{1}{(x+3)}$

$$\therefore \frac{1}{x}+\frac{1}{x+3}=\frac{1}{\frac{40}{13}}$$ 1

$$\Rightarrow \frac{(x+3)+x}{x(x+3)}=\frac{13}{40}$$

$$\Rightarrow \frac{2x+3}{x^2+3a}=\frac{13}{40}$$

$$\Rightarrow 13x^2+39x=80x+120$$ 1

$$\Rightarrow 13x^2-41x-120=0$$

$$\Rightarrow 13x^2-(65-24)x-120=0$$

$$\Rightarrow 13x^2-65x+24x-120=0$$

$$\Rightarrow 13x(x-5)+24(x-5)=0$$

$$\Rightarrow (x-5)(13x+24)=0$$

$$\Rightarrow x-5=0 \text{ or } 13x+24=0$$

$$\Rightarrow x=5 \text{ or } x=\frac{-24}{13}$$ 1

$$\Rightarrow x=5$$ $\frac{1}{2}$

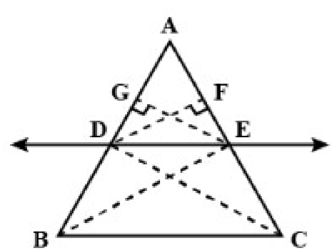
Thus, one pipe will take 5 mins and other will take {(5+3)=8} mins to fill the cistern.

33 (A)

Statement:

According to the BPT (Basic Proportionality Theorem), when a line is drawn parallel to one of the three sides of a triangle in such a way that it intersects the other two sides in distinct points, then the other two sides of the same triangle are divided into the same ratio. 1

Proof:



1

Given:

In ΔABC, DE || BC and AB and AC are intersected by DE at points D and E respectively. $\frac{1}{2}$

To prove:

AD / DB = AE / EC $\frac{1}{2}$

Construction:

Join BE and CD.

Draw:

EG ⊥ AB and DF ⊥ AC

Proof:

It is known that

$ar(\Delta ADE) = 1/2 \times AD \times EG$

$ar(\Delta DBE) = 1/2 \times DB \times EG$ $\frac{1}{2}$

Therefore, the ratio of these two can be computed as

$ar(\Delta ADE) / ar(\Delta DBE) = AD / DB$ . . . . (1) $\frac{1}{2}$

Similarly,

$ar(\Delta ADE) = ar(\Delta ADE) = 1/2 \times AE \times DF$

$ar(\Delta ECD) = 1/2 \times EC \times DF$

Therefore, the ratio of these two can be computed as $\frac{1}{2}$

$ar(\Delta ADE) / ar(\Delta ECD) = AE / EC$ . . . . (2)

Now,

ΔDBE and ΔECD are the same base DE and also between the same parallels i.e. DE and BC, we can get

$ar(\Delta DBE) = ar(\Delta ECD)$ . . . . (3)

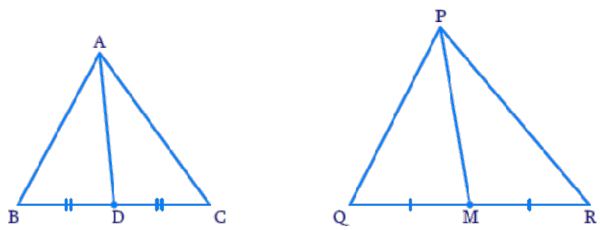
From three equations 1, 2, 3 it can be concluded that

$AD / DB = AE / EC$

Hence, the Basic Proportionality Theorem is proved. $\frac{1}{2}$

OR

(B)



Solution:

We know that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. This is referred to as SAS similarity criterion for two triangles.

In ΔABC and ΔPQR 1

$AB/PQ = BC/QR = AD/PM$ [given]

AD and PM are median of ΔABC and ΔPQR respectively

$\Rightarrow BD/QM = (BC/2)/(QR/2) = BC/QR$ 1

Now, in ΔABD and ΔPQM

$AB/PQ = BD/QM = AD/PM$

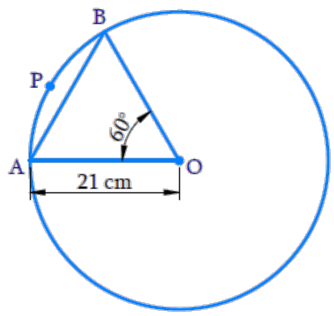
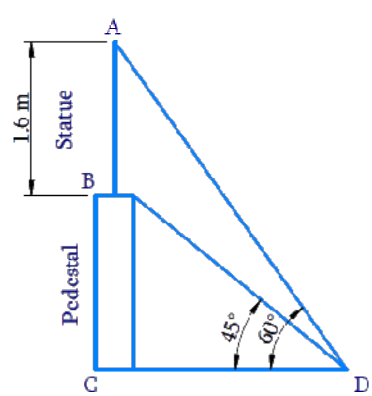
$\Rightarrow \Delta ABD \sim \Delta PQM$ [SSS criterion] 1

Now, in ΔABC and ΔPQR

$AB/PQ = BC/QR$ [given in the statement] 1

$\angle ABC = \angle PQR$ [$\because \Delta ABD \sim \Delta PQM$]

$\Rightarrow \Delta ABC \sim \Delta PQR$ [SAS criteion] 1

34	 Here, r = 21 cm, θ = 60° Visually it's clear from the figure that, Area of the segment APB = Area of sector AOPB - Area of ΔAOB (i) Length of the Arc, APB = θ/360° × 2πr = 60°/360° × 2 × 22/7 × 21 cm = 22 cm (ii) Area of the sector, AOBP = θ/360° x πr^2 = 60°/360° × 22/7 × 21 × 21 cm^2 = 231 cm^2 (iii) Area of ΔOAB = $\frac{\sqrt{3}}{4} \times a^2$ $= \frac{\sqrt{3}}{4} \times 21 \times 21$ $= \frac{441 \times 1.73}{4}$ $= \frac{762.93}{4}$ = 190.73 cm^2 Area of minor segment = (231 - 190.73) cm^2 = 40.27 cm^2 (iv) Area of Circle = $\pi r^2 = \frac{22}{7} \times 21 \times 21 = 66 \times 21 = 1386$ cm^2 Area of major segment = 1386 - 40.27 = 1345.73 cm^2	1 1 1 1 1
35	 Let the height of the pedestal be BC, the height of the statue, which stands on the top of the pedestal, be AB. D is the point on the ground from where the angles of elevation of the bottom B and the top A of the statue AB are 45° and 60° respectively.	1 $\frac{1}{2}$

	The distance of the point of observation D from the base of the pedestal is CD. Combined height of the pedestal and statue AC = AB + BC Trigonometric ratio involving sides AC, BC, CD, and ∠D (45° and 60°) is tan θ. In ΔBCD, tan 45° = BC/CD 1 = BC/CD Thus, BC = CD In ΔACD, tan 60° = AC/CD tan 60° = (AB + BC)/CD √3 = (1.6 + BC)/BC [Since BC = CD] √3 BC = 1.6 + BC √3 BC - BC = 1.6 BC (√3 - 1) = 1.6 BC = 1.6 × (√3 + 1)/(√3 - 1)(√3 + 1) = 1.6 (√3 + 1)/(3 - 1) = 1.6 (√3 + 1)/2 = 0.8 (√3 + 1) Height of pedestal BC = 0.8 (√3 + 1) m.	1 1 1 $\frac{1}{2}$
	SECTION E **Q. No. 36 to 38 are Case-Based Questions of 4 marks each.**	
36	(i) Area of floor $= \pi r^2 = \frac{22}{7} \times 28 \times 28 = 2464\ m^2$ Number of persons that can be accommodated in the tent $= \frac{2464}{17.6} = 140$ (ii) $l = \sqrt{r^2 + h^2} = \sqrt{1225} = 35$ m CSA of conical part = $\pi r l = \frac{22}{7} \times 28 \times 35 = 3080\ m^2$ (iii) (A) Required area of canvas $= \pi r l + 2\pi r h = \pi r(l + 2h) = \frac{22}{7} \times 28(35 + 2 \times 14) = 5544\ m^2$ ***OR*** (B) Volume of tent = Volume of cone + Volume of cylinder $= \frac{1}{3} \times \pi \times r^2 \times 21 + \pi r^2 \times 14 = \pi r^2(7 + 14) = \frac{22}{7} \times 28 \times 28 \times 21$ $= 51744\ m^3$	 1 1 1+1 1+1

37	(i) $\frac{5}{36}$ (ii) $\frac{6}{36} = \frac{1}{6}$ (iii) (A) $\frac{36}{36} = 1$ OR (B) $\frac{4}{36} = \frac{1}{9}$	**1** **1** **1+1** **1+1**
38	(i) 2 (ii) Parabola (iii) (A) $3x^2 - 16x - 12$ $= 3x^2 - 18x + 2x - 12$ = 3x (x-6) + 2 (x-6) = (x-6)(3x+2) Zeroes are 6 and $\frac{-2}{3}$ OR (B) Required polynomial is K[x^2 - (2+$\sqrt{3}$ + 2 - $\sqrt{3}$)x + 1 = K[x^2 - 4x + 1] Where K is a non zero real number.	**1** **1** **1+1** **1+1**